On Stability and Throughput for Multiple Access with Cooperation

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Abstract

In the wireless two-user multiple-access system with a single destination, the characterization of the stability region and the throughput region is revisited in the context of a packet-based "network-level" cooperation among the two users. Cooperation is achieved through the relaying of packets by the user with a better user-destination channel. Through queueing-theoretic techniques, the stability and throughput regions are characterized and compared under a centralized scheduled access and a random access scheme respectively. We conclude that the throughput region depends on the scheduling policies at the relay node, and may or may not be equal to the stability region, which is shown to be independent of the scheduling policies. By contrast, in the non-cooperative random access system, the stability region and the throughput region are proved to be identical. Another significant outcome is that the cooperative stability region under random access does not necessarily outer bound the non-cooperative stability region. Furthermore, when network coding is applied at the relay node as an alternative to simple retransmission, we show that there is no improvement either in the stability region or in the throughput region over plain store-and-forward routing.

Index Terms

Network-level cooperation, network coding, stability region, throughput region, queueing theory

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I. Introduction

The classical single-relay channel was first introduced by van der Meulen [1]; later, Cover and El Gamal considered the Shannon information capacity region of the classical relay channel in [2], and were able to determine it for the class of physically degraded channels. Subsequently, numerous papers have contributed to the understanding of the relay channel, including [3], [4], [5], et al. However, even for the single-relay channel, the Shannon information capacity remains unknown.

Recently, cooperative communication has received renewed attention as a powerful technique to combat fading and attenuation in wireless networks [6], [7], [8], [9], [10], [11], [12]. Cooperative diversity was realized by combining antennas belonging to different users, enabling intermediate users to relay information for other users as well as their own data. Most of this work exploited cooperative diversity at the physical layer and from the information theoretic point of view; as a result, performance metrics such as achievable rates, capacity, and outage probabilities were evaluated to determine the benefit of user cooperation.

A more recent approach exploited cooperation at the network level in a cognitive environment, where a pure relay node was introduced to forward packets from the source users [13]. In this setting, the relay was only allowed to forward a packet which was correctly decoded; the forwarded packet was an exact replica of the original packet. Accordingly, stability region and throughput region in terms of packet/slot were characterized through queueing-theoretic techniques. Lately, the pure relay assumption was dropped and the relay node was also allowed to be a source user of the network with its own data to be conveyed to the destination [14], [15], [16]. In [16], cooperation was performed by offering the users that have a better channel to the destination the option to relay packets from those that are farther afield. Under the erasure channel model which accounts for fading and attenuation, the impact of such cooperation strategy was evaluated with respect to the stability region for a tandem of N users, based on conflict-free scheduling only.

In this paper, we investigate a packet-based (or, so-called, network-level) cooperation in a wireless multiple-access system. The channel model and cooperation strategy are similar to those in [16], but this paper goes beyond [16] in several aspects:

1) Both scheduled access and random access are considered at the MAC layer. Because of

the formidable difficulty of stability analysis under random access with a large number of interacting queues, we restrict our analysis to the two-user case. It has been proved in [16] that under scheduled access, such cooperation leads to a larger stability region; however, we show that under random access, such cooperation does not necessarily yield a stability region superior to the non-cooperative stability region.

2) We develop performance analysis of the throughput region as well as the stability region, and hence, the relationship between these two regions in a multiple-access system is revisited, amplified, and explained. The relationship between these two regions has been extensively studied in the case of no cooperation, and it was established that these two regions are identical for the two-user case under a variety of channel and traffic models [17], [18], [19], [20], [21], but may also be different [17], [20], [21]. In this paper, we apply a class of scheduling policies for the transmission of the source packets and relayed packets at the relay node. We show that, the stability region is independent of the scheduling policies; however, the throughput region does depend on the scheduling policies, and is not necessarily identical to the stability region. This result rekindles interest in the relation between these two regions. In addition, we determine the optimal policy for maximizing the throughput region.
3) Network coding is performed at the relay node as an alternative to plain store-and-forward routing, and its impact on both the stability and throughput regions is evaluated. We conclude that network coding between the two traffic streams at the relay node leads to the same performance as plain store-and-forward routing.

A simpler version of our model was studied in [22], where the authors considered a linear tandem network in which each node was only connected to its two neighboring nodes via error-free links. However, this simple connectivity model does not accurately capture the physical-layer property of wireless channels. In [22], the authors did not consider the stability issue under random access; when they analyzed the throughput region, they assumed that all queues in the network are saturated which guarantees permanent availability of packets for transmission. However, even under heavy traffic, the queues of packets to be relayed are built by arrivals from the neighboring nodes and, hence, can be empty with positive probability. In this paper we characterize the throughput region by assuming that *only* the source queues at the user nodes are saturated, while the relay queue at the relay accepts bursty arrivals departing from the other

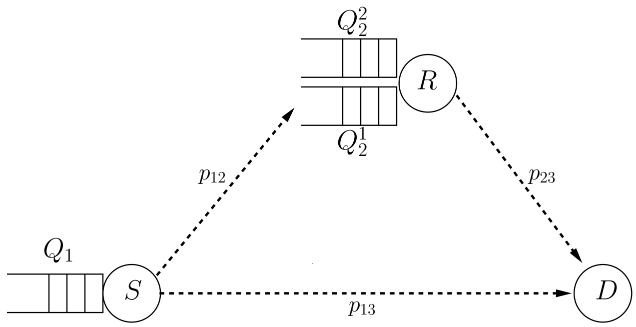

Fig. 1. The two-user multiple-access system with cooperation between the two source users S and R. D is the common destination. Channel reception probabilities are denoted above the channels.

user and, hence, need not be saturated. We show that by assuming that all queues are saturated, the actual throughput region is over-estimated.

The three-node cooperative network is the smallest and simplest non-trivial example that incorporates all the ingredients of a wireless network, namely, the wireless multicast advantage (which refers to the fact that a single transmission may reach multiple receivers), the "multiple-access" character of the channel, either under the collision regime model, whereby simultaneous transmissions attempted by two users cannot be successful, or the erasure channel model which captures fading and attenuation at the physical layer; and it incorporates the new concept of cooperation.

The rest of the paper is organized as follows: Section II describes the network model we consider throughout the paper. Section III and Section IV characterize and compare the stability and throughput regions under the scheduled access and random access scheme respectively. The performance with respect to network coding at the relay node is presented in Section V. Numerical results are shown in Section VI, and finally Section VII concludes the paper.

II. Network Model

We consider the slotted two-user multiple-access system shown in Fig. 1. Two source users, S and R, transmit unicast traffic to the common destination node D. The source nodes are equipped with buffers of infinite size to store incoming packets, and the transmission of each packet takes exactly one time slot. Nodes are ordered according to Fig. 1 so that R has a better source-destination channel than S. The channel quality is reflected by the channel reception probability, which is equal to the probability that the Signal-to-Noise Ratio (SNR) exceeds a certain threshold. This model captures the effects of fading and attenuation at the physical layer. If a packet is not fully decoded, the receiver simply drops the packet[1]. We consider omnidirectional transmissions so that a transmission may reach successfully multiple nodes in the network, as long as the received SNR exceeds the threshold. Along with the binary input and output assumption of the channel process, we also assume that the system is operating on slow Rayleigh fading, so that the channel fading coefficients remain constant during one time slot, and the channel processes are independent of each other and i.i.d. from one slot to another. Therefore, the quality of each of the three channels in the network can be represented by a reception probability; specifically, p_{13} and p_{23} denote the source-destination channel reception probabilities from the sources S and R to the destination respectively; and p_{12} denotes the inter-user channel reception probability between S and R. By using these notations, the fact that source user R has a better channel to the destination (than S) can be mathematically expressed as $p_{23} > p_{13}$. The reception probabilities p_{13}, p_{23} and p_{12} are determined by parameters such as the network topology, transmission power, targeted bit-error rate, etc., and cannot be altered once these parameters are fixed.

Since user R has a better channel to the destination, it is preferable to let R relay packets for S, if those packets transmitted by S are not decoded by the destination but decoded by R. Based on this consideration, the *cooperation strategy* used in this paper is stated as follows: when node S transmits a packet, if the destination D decodes the packet successfully, it sends back an acknowledgement (ACK) and the packet exits the system; otherwise, if D doesn't decode the packet but R decodes the packet, R sends back an ACK and keeps the packet in its queue for retransmission; upon receiving the ACK from R, S will drop the packet in this case; if neither R nor D decodes the packet, the packet will remain in S's queue for retransmission. For R's own packets, R is responsible for delivering them to the destination without help from S. As usual, ACKs are assumed to be error-free and instantaneous, and broadcast to the whole network over a separate control channel with negligible bandwidth.

With this form of cooperation, there are two "virtual" queues at node R, as depicted in Fig. 1: the queue Q_2^1 stores packets relayed from S, and Q_2^2 stores R's own packets. We refer to Q_1 and Q_2^2 as the source queues, and Q_2^1 as the relay queue. This cooperative three-node network model resembles the classical relay channel in information-theoretical studies; in this paper, we will focus our attention on investigating the performance metrics of the stability region and

[1]This is a simplified assumption that can be modified if enhanced physical layer techniques are superposed such as incremental redundancy, Alamouti coding, etc.

throughput region.

The stability region and throughput region correspond to either bursty traffic arrivals or saturated source queues respectively. In the non-saturated system, packets are generated at nodes S and R independently according to Bernoulli processes, with rates λ_1 and λ_2 respectively. We study the stability region which is defined as the union of all (λ_1, λ_2) such that all queues in the network remain bounded[2]. An important tool to determine stability is Loynes' Theorem [23], which states that if the arrival and service processes in a queueing system are strictly stationary and ergodic, the queue is stable if and only if the average arrival rate is strictly less than the average service rate[3]. In the saturated system, we assume permanent availability of source packets at the two source queues Q_1 and Q_2^2; accordingly, we study the throughput region which is the union of all achievable throughput rate pairs (λ_1, λ_2), where λ_1 and λ_2 stand for the throughput rates for S and R respectively. We should note here that the relay queue Q_2^1 at node R is formed by arrivals from S according to a stationary process, and hence, cannot be assumed to be saturated.

At the MAC layer, we separately consider a centralized scheme and a fully distributed scheme, namely, scheduled access and random access. The two medium access schemes will be described and analyzed in Section III and Section IV respectively.

We close this paper by considering network coding at the relay node. With two "virtual" queues available at the relay node R, the latter has the freedom to operate across the packets it relays and its own packets instead of simply forwarding each individual packet separately. We study linear network coding, which is simple to implement and analyze. With linear network coding, the relay node R either transmits the linear combination of two packets, one from Q_2^1 and one from Q_2^2, if both queues are non-empty; or transmits an individual uncoded packet if only one queue is non-empty.

III. Scheduled Access

Under scheduled access, the source nodes S and R access the channel at disjoint fractions of time ω_1 and ω_2 respectively, where $0 \leq \omega_1 \leq 1$, $0 \leq \omega_2 \leq 1$. The set of all feasible time allocation

[2] The strict definition of stability is provided in [23].

[3] The stability at the boundary with equivalence between the arrival rate and service rate is hard to determine, and is out of the scope of this paper. The comparison between the stability region and the throughput region is undertaken for the inner points only, hence bypasses the boundary points.

satisfies: $\sum_{i=1}^{2} \omega_i \leq 1$. The two source nodes are activated to transmit at different time slots, so there is no contention. For each allocation (ω_1, ω_2), we can find a corresponding stability region and throughput region; then, we will take the union of such regions over all possible (ω_1, ω_2) such that $\sum_{i=1}^{2} \omega_i \leq 1$. Therefore, for any point (λ_1, λ_2) inside the thus obtained stability region, there exist time allocations (ω_1, ω_2) such that the queues in the network remain stable; similarly, for any point (λ_1, λ_2) in the throughput region, there exist time allocations (ω_1, ω_2) such that the throughput rates (λ_1, λ_2) can be achieved.

In this multiple-access cooperative system, there are two "virtual" queues Q_2^1 and Q_2^2 at node R, one stores packets relayed from S, and the other stores R's own packets. Therefore, when it is R's turn to transmit a packet, R can choose to transmit a packet from one of the two queues; the scheduling policy, that is, the policy for deciding which of the two queues to serve, may affect the (stable) throughput achieved by the two source nodes individually. We consider a class of scheduling policies for node R in deciding between Q_2^1 and Q_2^2: when both Q_2^1 and Q_2^2 are non-empty, R selects a packet from Q_2^1 with probability β, and a packet from Q_2^2 with probability $1-\beta$ (for $0 \leq \beta \leq 1$); when only one of the two queues is non-empty, R selects a packet from that queue. If we set $\beta = 0$, it corresponds to the case which assigns higher priority to Q_2^2; if we set $\beta = 1$, it corresponds to the case which assigns higher priority to Q_2^1. Putting together these two special cases with the general class of policies, the three scheduling policies we consider at the relay node are described as follows:

1) The source queue Q_2^2 is given priority; that is, only when it is empty, will R transmit a packet from the relay queue Q_2^1 ($\beta = 0$).
2) The relay queue Q_2^1 is given priority; that is, only when it is empty, will R serve a packet from the source queue Q_2^2 ($\beta = 1$).
3) When both queues Q_2^1 and Q_2^2 are non-empty, R selects a packet from Q_2^1 with probability β, and a packet from Q_2^2 with probability $1-\beta$, for $0 \leq \beta \leq 1$; when only one of the two queues is non-empty, R selects a packet from that queue.

Note that under policy 3, we will vary over $\beta \in [0, 1]$ to obtain the stability and throughput regions; the thus obtained regions are compared with the corresponding regions under policies 1 and 2.

A. Stability Analysis under Scheduled Access

Let the arrival processes at the two source nodes be independent Bernoulli processes. The stability region under scheduled access for a tandem of N users was obtained in [16]; an important observation from the proof in that analysis is that the resulting region is independent of the scheduling policies at the relay node. In addition, the stability region of the cooperative system strictly contains the stability region when cooperation is not used. Specializing the results from [16], for $N = 2$, we obtain:

Theorem 1: For any scheduling policy adopted by node R, the stability region under scheduled access in the two-user multiple-access system with cooperation is given by:

$$L_{\text{SR}} = \left\{ (\lambda_1, \lambda_2) : \frac{(p_{12} + p_{23} - p_{12}p_{13})\lambda_1}{p_{23}\,(p_{12} + p_{13} - p_{12}p_{13})} + \frac{\lambda_2}{p_{23}} < 1 \right\} \tag{1}$$

B. Throughput Analysis under Scheduled Access

Next, we consider the throughput region, that is, the region of rates that are feasible when we assume that the two source queues Q_1 and Q_2^2 are saturated. We denote by L_i the throughput region when the scheduling policy i ($i \in \{1, 2, 3\}$) is adopted. Note that even in the saturated case, the relay queue Q_2^1 at node R is built by the arrivals from Q_1, so it is non-saturated and can be empty at times. In [22], the authors considered the same scenario but neglected the bursty nature of the traffic relayed to the queue Q_2^1. We will show later that neglecting this crucial aspect of internode traffic leads to an overestimate of the real throughput region.

Under the assumption that Q_2^2 is always saturated, the scheduling policy 1 in which Q_2^2 is given priority implies that the relay queue Q_2^1 is never served by node R.

Theorem 2: Under scheduling policy 1, the throughput region reduces to the throughput region of the non-cooperative system, which is:

$$L_1 = \left\{ (\lambda_1, \lambda_2) : \frac{\lambda_1}{p_{13}} + \frac{\lambda_2}{p_{23}} < 1 \right\} \tag{2}$$

This is a strict subset of the stability region given by Eq. (1), that is, $L_1 \subset L_{\text{SR}}$.

Proof: The source queue Q_2^2 is saturated and never empties, so R always selects a packet from Q_2^2 to transmit when it is R's slot. This reduces to the non-cooperative system as node R

never serves the packets relayed from S. The throughput rates are given by:

$$\lambda_1 = \omega_1 p_{13} \qquad\qquad \lambda_2 = \omega_2 p_{23} \tag{3}$$

If we take the union over all (ω_1, ω_2) such that $\omega_1 + \omega_2 \leq 1$, we obtain the throughput region as shown in Eq. (2). A simple comparison will find that the throughput region under policy 1 is strictly inferior to the stability region given by Eq. (1). ■

Similarly, as Q_2^2 never empties, the scheduling policy 3 becomes: when Q_2^1 is non-empty, R selects a packet from Q_2^1 with probability β, and a packet from Q_2^2 with probability $1 - \beta$, for $0 \leq \beta \leq 1$; when Q_2^1 is empty, R selects a packet from Q_2^2. The throughput analysis under policy 2 and policy 3 are carried out, and the results lead to:

Theorem 3: Under scheduling policies 2 and 3, the throughput regions are the same and are *identical* to the stability region given by Eq. (1) of Theorem 1. So we have: $L_2 = L_3 = L_{\text{SR}}$.

Proof: Under both policies 2 and 3, the throughput rate for source S consist of two parts: the packets delivered to the destination by S itself, and those relayed by R. The throughput rate contributed by S itself is easily seen to be $\lambda_1^S = \omega_1 p_{13}$, since a packet is delivered from S to the destination if and only if the time slot is assigned to S and the packet is successfully decoded by the destination, which has the probability of $\omega_1 p_{13}$; we need further to calculate the throughput rate contributed by the relaying, that is, the throughput rate contributed by packet delivery from the queue Q_2^1.

As we have clarified before, Q_2^1 is built by arrivals from source S, and hence, can be empty with positive probability. Therefore, the behavior of Q_2^1 is determined by both its arrival process and its service process. A packet will arrive at Q_2^1 from node S if and only if the following three events happen together: first, the time slot is assigned to S, so S transmits a packet; second, the destination doesn't decode that packet; and third, the relay node R decodes that packet successfully. These three events are independent, so the expected value of the arrival process to the queue Q_2^1 is:

$$\lambda_{Q_2^1} = \omega_1 p_{12}(1 - p_{13}) \tag{4}$$

Under policy 2, Q_2^1 is given the priority over Q_2^2, and the packet is delivered to the destination from Q_2^1 if and only if it is node R's time slot, and the destination successfully decodes the transmitted packet. So the average service rate of Q_2^1 is calculated to be

$$\mu_{Q_2^1} = \omega_2 p_{23} \tag{5}$$

By Loynes' Theorem, if the arrival and service processes of a queue are strictly stationary and ergodic, the queue is stable if and only if the average arrival rate is strictly less than the average service rate, and furthermore, the departure rate is equal to the arrival rate. As a result, one and only one, of the following two cases will happen:

1) if $\omega_1 p_{12}(1-p_{13}) < \omega_2 p_{23}$, the queue Q_2^1 is stable, and the throughput rate for S contributed by the relaying is $\lambda_1^R = \omega_1 p_{12}(1-p_{13})$. Therefore, the total throughput rate for S is

$$\lambda_1 = \lambda_1^S + \lambda_1^R = \omega_1(p_{13} + p_{12}(1-p_{13})) \tag{6}$$

   In this case, Q_2^1 is empty with probability $1-\omega_1 p_{12}(1-p_{13})/\omega_2 p_{23}$ as it is simply a discrete-time M/M/1 queue. According to policy 2, Q_2^2 is successfully served if it is node R's time slot and Q_2^1 is empty, and the packet transmitted from Q_2^2 is decoded by the destination. So the average service rate that Q_2^2 receives is given by

$$\mu_{Q_2^2} = \omega_2 p_{23}\left(1 - \frac{\omega_1 p_{12}(1-p_{13})}{\omega_2 p_{23}}\right) = \omega_2 p_{23} - \omega_1 p_{12}(1-p_{13}) \tag{7}$$

   This is the actual throughput rate for user R. So we obtain the throughput rates as:

$$\lambda_1 = \omega_1(p_{13} + p_{12}(1-p_{13})) \tag{8}$$

$$\lambda_2 = \omega_2 p_{23} - \omega_1 p_{12}(1-p_{13}) \tag{9}$$

2) if $\omega_1 p_{12}(1-p_{13}) \geq \omega_2 p_{23}$, Q_2^1 is unstable and will grow without bound. In this case, the throughput rate for S relayed by R is equal to the service rate of Q_2^1, which is $\omega_2 p_{23}$; besides, Q_2^1 is empty with probability zero, so Q_2^2 is served with probability zero as it is assigned lower priority, and the throughput rate for R is zero. Therefore, the throughput rates are given by:

$$\lambda_1 = \omega_1 p_{13} + \omega_2 p_{23} \tag{10}$$

$$\lambda_2 = 0 \tag{11}$$

By taking the union of rates over all feasible (ω_1, ω_2) such that $\sum_{i=1}^{2} \omega_i \leq 1$, we obtain that the actual throughput region under policy 2 is characterized by Eq. (1).

Under policy 3, given fixed (ω_1, ω_2), a similar analysis will lead to the throughput rates as functions of ω_1, ω_2 and β, given by the following:

1) if $\omega_1 p_{12}(1 - p_{13}) < \beta\omega_2 p_{23}$, the throughput rates are given by:

$$\lambda_1 = \omega_1(p_{13} + p_{12}(1 - p_{13})) \tag{12}$$

$$\lambda_2 = \omega_2 p_{23} - \omega_1 p_{12}(1 - p_{13}) \tag{13}$$

2) if $\omega_1 p_{12}(1 - p_{13}) \geq \beta\omega_2 p_{23}$, the throughput rates are given by:

$$\lambda_1 = \omega_1 p_{13} + \beta\omega_2 p_{23} \tag{14}$$

$$\lambda_2 = (1 - \beta)\omega_2 p_{23} \tag{15}$$

By taking the closure of the throughput rates over all (ω_1, ω_2) such that $\sum_{i=1}^{2} \omega_i \leq 1$ and $\beta \in [0, 1]$, the throughput region under policy 3 is also characterized by Eq. (1). ■

The above analysis reveals that the arrival and service processes of the non-saturated queue Q_2^1 play a critical role in determining the throughput region under policy 2 and policy 3. If we instead assume that all queues are saturated including the relay queue Q_2^1, as was done in [22], the maximum throughput region is achieved by activating node R all the time, that is, by assigning $\omega_1 = 0$ and $\omega_2 = 1$. Node R switches between Q_2^1 and Q_2^2, and the throughput region is given by $L' = \{ (\lambda_1, \lambda_2) : \lambda_1 + \lambda_2 < p_{23} \}$. This strictly contains the region expressed in Eq. (1). So by assuming all queues saturated, we will over-estimate the throughput region, and the corresponding result provided in [22] turns out to be a strict outer bound.

C. Optimal Policy for the Throughput Region

As shown above, the throughput region under policy 3 obtained by taking the union of throughput rates for all possible $\beta \in [0, 1]$, where β is the probability to select a packet from the relay queue Q_2^1, is the same as the throughput region under policy 2 where Q_2^1 is always given the priority over Q_2^2. In addition, the throughput regions under policy 2 and policy 3 strictly bound the throughput region under policy 1. These two observations suffice to conclude that, the scheduling policy which assigns higher priority to the relay queue Q_2^1 is optimal in the sense that it achieves the maximum throughput region among the class of stationary scheduling policies considered at the relay node.

IV. Random Access

Due to the difficulty of global coordination for scheduled access in ad hoc networks, random access is an attractive and simple alternative that is implemented in a fully distributed fashion. In this setting, when source node S (or R) is backlogged, it transmits a packet with probability q_1 (or q_2) independently of any other event. In this way, if both S and R decide to transmit in the same time slot, a collision will occur which we assume leads to failure of both transmissions. We also do not allow simultaneous transmission and reception, and hence, when R is transmitting, it cannot receive a packet transmitted by S.

We are interested in characterizing the stability and throughput regions corresponding to each of the three scheduling policies described in Section III, upon which we can reveal the relation between the two regions for random access. Likewise, the stability region is obtained by taking the set of all arrival rates (λ_1, λ_2) such that there exist transmission probabilities (q_1, q_2) which stabilize all queues; the throughput region is defined by the set of all throughput rates (λ_1, λ_2) for which there exist (q_1, q_2) which can achieve them.

A. Stability Analysis under Random Access

Stability analysis in random-access system is known to be notoriously difficult due to the interaction among the queues. So far, the stability region of the slotted ALOHA system is known for no more than three users. Even in the case of two users without cooperation (the two-user ALOHA system), the stability analysis is non-trivial.

A useful tool for finding the stability region of a random-access system is the stochastic dominance approach introduced in [17]. The essential part of this approach lies as follows: first, one tries to construct appropriate dominant system in which one queue can be decoupled from the other, which makes the stability analysis tractable; second, one tries to prove that the dominant system and the original system behave identically at the boundary of the stability region. Using the stochastic dominance approach we derive the stability region, and the main result is summarized in the following theorem:

Theorem 4: The stability region under random access is independent of the scheduling policies at the relay node R, and is characterized by:

$$\Re_{\text{SR}} = \left\{ (\lambda_1, \lambda_2) : \sqrt{\frac{\lambda_1}{p_{12} + p_{13} - p_{12}p_{13}}} + \sqrt{\frac{p_{12}(1 - p_{13})\lambda_1}{p_{23}(p_{12} + p_{13} - p_{12}p_{13})} + \frac{\lambda_2}{p_{23}}} < 1 \right\} \tag{16}$$

Proof: See Appendix A. ■

B. Comparison of the Cooperative and Non-cooperative Stability Regions

Under scheduled access, we have proved in [16] that the cooperative stability region strictly contains the stability region when cooperation is not used. Under random access, the stability region without cooperation is known to be

$$\Re_{\text{SR}}^{\text{nocoop}} = \left\{ (\lambda_1, \lambda_2) : \sqrt{\frac{\lambda_1}{p_{13}}} + \sqrt{\frac{\lambda_2}{p_{23}}} < 1 \right\} \tag{17}$$

From Eq. (16), the maximum stabilizable rate for source S with cooperation is $\frac{p_{23}(p_{12}+p_{13}-p_{12}p_{13})}{(\sqrt{p_{12}(1-p_{13})}+\sqrt{p_{23}})^2}$, this quantity can be greater than, equal to or less than p_{13}, which is the maximum stabilizable rate for S in the non-cooperative system. By comparing the two regions expressed in Eq. (16) and Eq. (17), we can easily find that the relation between the two regions depends on the relation between these two quantities that are functions of the system parameters. Specifically,

1) If

$$\frac{p_{23}(p_{12} + p_{13} - p_{12}p_{13})}{(\sqrt{p_{12}(1 - p_{13})} + \sqrt{p_{23}})^2} \geq p_{13} \tag{18}$$

the stability region of the cooperative system under random access strictly contains the stability region of the non-cooperative system. The relation between these two regions in this case is illustrated in Fig. 2(a);

2) If

$$\frac{p_{23}(p_{12} + p_{13} - p_{12}p_{13})}{(\sqrt{p_{12}(1 - p_{13})} + \sqrt{p_{23}})^2} < p_{13} \tag{19}$$

the stability region of the cooperative system and the stability region of the non-cooperative system overlap, but neither one properly contains the other. The boundaries of the two stability regions have two intersection points, which are $(\lambda_1', \lambda_2') = (0, p_{23})$ on the y-axis,

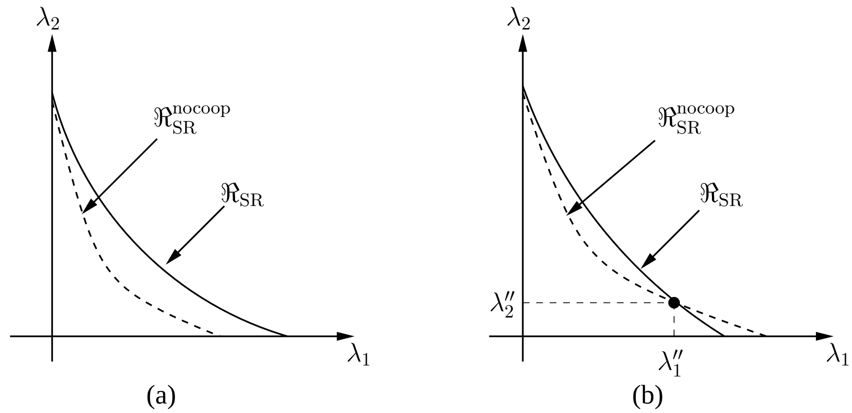

Fig. 2. Comparison of the stability regions for the cooperative and non-cooperative systems under random access. (a) Condition (18) holds, $\Re_{\rm SR} \supset \Re_{\rm SR}^{\rm nocoop}$. (b) Condition (19) holds, $\Re_{\rm SR}$ and $\Re_{\rm SR}^{\rm nocoop}$ cross at $(\lambda_1'', \lambda_2'')$.

and $(\lambda_1'', \lambda_2'') = (\alpha p_{13}, (1-\sqrt{\alpha})^2 p_{23})$, with $\alpha = \frac{4\left(\sqrt{p_{12}-p_{12}p_{13}+p_{13}}-\sqrt{p_{13}}\right)^2 (p_{12}-p_{12}p_{13}+p_{13})p_{23}^2}{(p_{12}-p_{12}p_{13})^2(p_{13}+p_{23})^2}$. In the subset constrained by $\lambda_1 \leq \lambda_1''$ (or equivalently, $\lambda_2 \geq \lambda_2''$), the cooperative stability region strictly contains the non-cooperative stability region; in the subset constrained by $\lambda_1 > \lambda_1''$ (or equivalently, $\lambda_2 < \lambda_2''$), the non-cooperative stability region strictly contains the cooperative stability region. This relationship is illustrated in Fig. 2(b).

Condition (18) implies that p_{23} is considerably larger than p_{13}, that is, the channel condition from R to D is sufficiently better than the channel condition from S to D; By contrast, condition (19) implies that the R to D channel quality is better than the S to D one, but not sufficiently so. When cooperation is used, some of S's packets will enter R's queue for retransmission, and the transmission of these packets from R will collide with the transmission from S, if S decides to transmit at the same time. Therefore, we observe that two forces are at work:

(a) For one thing, some of S's packets will be relayed by R, which has the better user-destination channel; this is a positive effect.

(b) However, the addition of such an arrival rate to R requires R to push more packets out of its queue if it wants to remain stable; and under random access, the simultaneous transmissions of R and S will cause destructive collision and both transmissions fail. This is a negative effect.

Based on these two effects, under condition (19) where the R to D channel is not sufficiently better than the S-D channel, the following happens: when λ_1 is very small, the cooperative system outperforms the non-cooperative system in terms of stability region. In this range, effect (a) plays the major role. As we increase λ_1 until it exceeds a certain threshold value λ_1'', the cooperative stability region falls inside the non-cooperative stability region. This is because, as λ_1 increases, the external arrival rate from S to R increases as well, and the occurrence of collision increases. As a result, effect (b) becomes dominant and the cooperation strategy actually degrades the performance.

C. Throughput Analysis under Random Access

Under the assumption that the source queues Q_1 and Q_2^2 are saturated, we investigate the throughput regions with respect to each of the three scheduling policies at node R as described in Section III. We denote by $\Re_i$ the throughput region when the scheduling policy i ($i \in \{1, 2, 3\}$) is adopted.

Theorem 5: Under scheduling policy 1, the throughput region is given by:

$$\Re_1 = \left\{ (\lambda_1, \lambda_2) : \sqrt{\frac{\lambda_1}{p_{13}}} + \sqrt{\frac{\lambda_2}{p_{23}}} < 1 \right\} \tag{20}$$

Proof: As we have argued in the proof of Theorem 2 in Section III-B, the saturated system in which the source traffic at the relay node is given priority reduces to the non-cooperative system. Under random access, this system behaves the same as the slotted ALOHA system with an erasure channel, and the throughput region is known to be given by Eq. (20), which is also the non-cooperative stability region. ■

Theorem 6: Under scheduling policy 2 and policy 3, the throughput regions $\Re_2$ and $\Re_3$ are given by:

1) When the condition in Eq. (18) is satisfied, the throughput regions under policy 2 and policy 3 are the same, and are identical to the cooperative stability region as given by Eq. (16) in Theorem 4. That is,

$$\Re_2 = \Re_3 = \Re_{\text{SR}} \tag{21}$$

2) When the condition in Eq. (19) is satisfied, the throughput region under policy 2 is given by the union of two regions, specifically,

$$\Re_2 = \Re_{\text{SR}} \bigcup \Re' \tag{22}$$

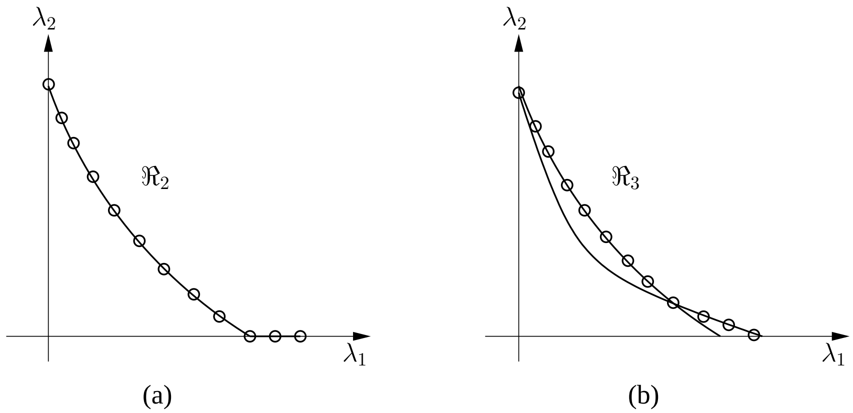

Fig. 3. Boundaries of the throughput regions under policy 2 and policy 3 for random access when condition (19) holds. (a) $\Re_2$, throughput region under policy 2. (b) $\Re_3$, throughput region under policy 3.

where $\Re_{\text{SR}}$ is the cooperative stability region as shown in Eq. (16), and $\Re'$ is defined by:

$$\Re' = \left\{ (\lambda_1, \lambda_2) : \begin{array}{l} \frac{p_{23}(p_{12}+p_{13}-p_{12}p_{13})}{(\sqrt{p_{12}(1-p_{13})}+\sqrt{p_{23}})^2} \leq \lambda_1 \leq p_{13} \\ \lambda_2 = 0 \end{array} \right\} \tag{23}$$

A diagram of the boundary of $\Re_2$ given by these equations is shown in Fig. 3(a). Finally, the throughput region under policy 3 is given by:

$$\Re_3 = \Re_{\text{SR}} \bigcup \Re_1 \tag{24}$$

where $\Re_{\text{SR}}$ and $\Re_1$ are given in Eq. (16) and Eq. (20) respectively. A diagram showing the boundary of $\Re_3$ is depicted in Fig. 3(b). In this case, the relationship between the cooperative stability region and the throughput regions under policies 2 and 3 is:

$$\Re_3 \supset \Re_2 \supset \Re_{\text{SR}} \tag{25}$$

Proof: The throughput rates for nodes S and R under policy 2 and policy 3 can be computed in the same way as in the proof of Theorem 3 in Section III-B. For fixed (q_1, q_2), under policy 2, we can obtain that

1) If

$$q_1(1-q_2)p_{12}(1-p_{13}) < q_2(1-q_1)p_{23} \tag{26}$$

the throughput rates are given by:

$$\lambda_1 = q_1(1-q_2)(p_{12} - p_{12}p_{13} + p_{13})$$
$$\lambda_2 = q_2(1-q_1)p_{23} - q_1(1-q_2)p_{12}(1-p_{13}) \tag{27}$$

2) If

$$q_1(1-q_2)p_{12}(1-p_{13}) \geq q_2(1-q_1)p_{23} \tag{28}$$

the throughput rates are given by:

$$\lambda_1 = q_1(1-q_2)p_{13} + q_2(1-q_1)p_{23}$$
$$\lambda_2 = 0 \tag{29}$$

The throughput region is the union of all these rates over $(q_1, q_2) \in [0,1]^2$. When the condition written in Eq. (26) is satisfied, by following the same procedure as in the proof of Theorem 4 which solves a constrained optimization problem, we can obtain that the throughput region is given by

$$\sqrt{\frac{\lambda_1 p_{12}(1-p_{13})}{p_{23}(p_{12}+p_{13}-p_{12}p_{13})} + \frac{\lambda_2}{p_{23}}} + \sqrt{\frac{\lambda_1}{p_{12}+p_{13}-p_{12}p_{13}}} < 1 \tag{30}$$

On the other hand, when the condition in Eq. (28) holds, the throughput region can be shown to be

$$\begin{cases} 0 \leq \lambda_1 \leq \max\left(p_{13}, \frac{p_{23}(p_{12}+p_{13}-p_{12}p_{13})}{\left(\sqrt{p_{12}(1-p_{13})}+\sqrt{p_{23}}\right)^2}\right) \\ \lambda_2 = 0 \end{cases} \tag{31}$$

The throughput region under policy 2 is the union of these two sub-regions; depending on the relation between the quantities p_{13} and $\frac{p_{23}(p_{12}+p_{13}-p_{12}p_{13})}{\left(\sqrt{p_{12}(1-p_{13})}+\sqrt{p_{23}}\right)^2}$, the throughput region is as described in Theorem 6.

Under policy 3, by following a similar analysis, we can obtain that for fixed (q_1, q_2) and fixed β, we have:

1) If $q_1(1-q_2)p_{12}(1-p_{13}) < \beta q_2(1-q_1)p_{23}$, the throughput rates are given by:

$$\begin{aligned} \lambda_1 &= q_1(1-q_2)(p_{12}-p_{12}p_{13}+p_{13}) \\ \lambda_2 &= q_2(1-q_1)p_{23} - q_1(1-q_2)p_{12}(1-p_{13}) \end{aligned} \tag{32}$$

2) If $q_1(1-q_2)p_{12}(1-p_{13}) \geq \beta q_2(1-q_1)p_{23}$, the throughput rates are given by:

$$\begin{aligned} \lambda_1 &= q_1(1-q_2)p_{13} + \beta q_2(1-q_1)p_{23} \\ \lambda_2 &= (1-\beta)q_2(1-q_1)p_{23} \end{aligned} \tag{33}$$

After taking the union over $(q_1, q_2) \in [0,1]^2$ and $\beta \in [0,1]$, the throughput region under policy 3 can be shown to be given by Theorem 6.

Finally, a simple comparison between $\Re_2$, $\Re_3$ and $\Re_{\mathrm{SR}}$ yields the relations between these regions as shown in Eq. (21) and Eq. (25), under condition (18) and condition (19) respectively. ■

D. Optimal Policy for the Throughput Region

From Theorems 5 and 6, we learn that if condition (18) is satisfied, the throughput region under policy 2 coincides with the throughput region under policy 3, which strictly bounds the throughput region under policy 1. That is, $\Re_2 = \Re_3 \supset \Re_1$. Hence, in this case, the scheduling policy which always assigns higher priority to the relay queue Q_2^1 (corresponding to $\beta = 1$) is optimal in achieving the maximum throughput region among the stationary scheduling policies considered at the relay node. The optimal scheduling policy under condition (18) is shown in Fig. 4(a).

Otherwise, that is, if condition (19) is satisfied, the optimal scheduling policy is not the same for the entire range. In this case, a simple comparison will find that the maximum throughput region is given by the union of two regions: the throughput region under policy 1 and the throughput region under policy 2. That is, $\Re_{\mathrm{TR}}^{\max} = \Re_1 \bigcup \Re_2$. In addition to the two points on the x-axis and y-axis, the boundaries of the two throughput regions intersect at the point $(\lambda_1^*, \lambda_2^*) = (\alpha p_{13}, (1-\sqrt{\alpha})^2 p_{23})$, with $\alpha = \frac{4\left(\sqrt{p_{12}-p_{12}p_{13}+p_{13}}-\sqrt{p_{13}}\right)^2 (p_{12}-p_{12}p_{13}+p_{13})p_{23}^2}{(p_{12}-p_{12}p_{13})^2(p_{13}+p_{23})^2}$. In the subset constrained by $\lambda_1 \leq \lambda_1^*$ (or equivalently, $\lambda_2 \geq \lambda_2^*$), $\Re_2$ strictly contains $\Re_1$; in the subset constrained by $\lambda_1 > \lambda_1^*$ (or equivalently, $\lambda_2 < \lambda_2^*$), $\Re_1$ strictly contains $\Re_2$. Therefore, among

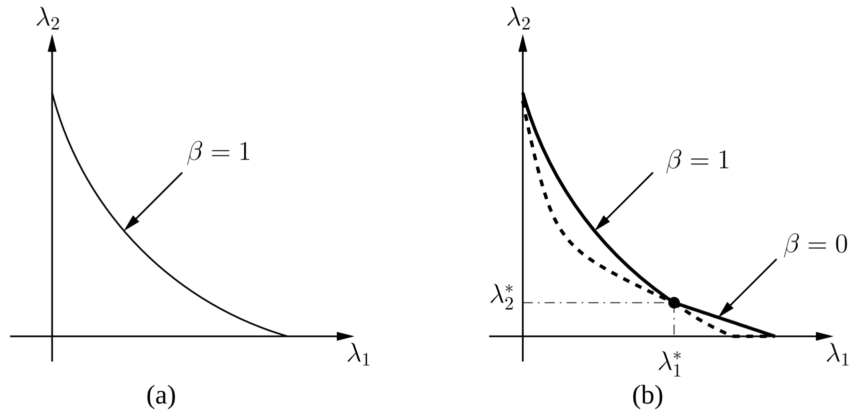

Fig. 4. The optimal scheduling policies that maximize the throughput region for random access under (a) condition (18), and (b) condition (19).

the class of scheduling policies considered, the optimal policy at the relay node in achieving the maximum throughput region is described as follows: if we want to achieve the throughput rates for $\lambda_1 \leq \lambda_1^*$ (or equivalently, $\lambda_2 \geq \lambda_2^*$), the optimal scheduling policy is the policy which assigns higher priority to the relay queue Q_2^1 ($\beta = 1$); if we want to achieve the throughput rates for $\lambda_1 > \lambda_1^*$ (or equivalently, $\lambda_2 < \lambda_2^*$), the optimal scheduling policy is the one which assigns higher priority to relay's source queue Q_2^2 ($\beta = 0$). The latter case corresponds to the scenario of the non-cooperative operation, as Q_2^2 never empties and Q_2^1 will never be served. The optimal scheduling policy under condition (19) is shown in Fig. 4(b).

V. Network Coding at the Relay Node

The idea of Network Coding was first introduced in [24]. It allows nodes to perform operations on the bits inside the packets that are received from different sources. In [25] where a configuration similar to the one we are considering here was analyzed. It was shown that certain coding schemes at the relay node achieve the *min-cut capacity*; among the coding schemes at the relay node, greedy random linear coding is proved to be rate-optimal when the channels are noiseless, provided that the relay node can transmit and receive information simultaneously.

With both relayed packets and source packets at the relay node R, we investigate the performance of network coding when it is applied on these two streams of packets, and compare it to the simple retransmission scheme. When node R decides to transmit, network coding is performed in the following fashion: if both queues Q_2^1 and Q_2^2 are non-empty, node R transmits a random linear combination of two packets, one from Q_2^1, the other from Q_2^2; if only one queue is non-empty, node R transmits an uncoded packet from that queue. The random linear combination is assumed to be over a sufficiently large finite field, so the coefficient vectors of the same set of packets generated in different time slots are linearly independent with probability approaching one. It is easy to show the following result:

Theorem 7: The superposition of network coding over cooperative relaying does *not* introduce additional performance gains for either the stability region or the throughput region, under either scheduled access or random access. The performance of network coding at the relay node R in this three-node packet-erasure network is the same as that of plain store-and-forward routing.

Proof: Consider the coded packet $\alpha_1 A + \alpha_2 B$, where packets A and B are from the queues Q_2^1 and Q_2^2 respectively, and coefficients α_1, α_2 are randomly generated in each time slot. Since both A and B are new to the destination, the latter needs to successfully receive two such encoded packets to decode both A and B. Therefore, we can view each encoded packet as a new uncoded packet and it follows that network coding yields the same performance as the plain store-and-forward routing in this case. ■

This conclusion can be extended to the case where the relay node is allowed to perform random linear network coding on K packets, for any $K \geq 2$; or even the greedy random coding scheme which allows the relay to transmit a linear combination of all packets in its queue at each time. There will be no performance gain over the plain store-and-forward routing in either the stability or throughput region, by following the same argument. Furthermore, if we take into account the non-zero probability of linear dependence between the coding vectors applied to the same set of combined packets, the destination is required to receive on average more than two encoded packets in order to decode two individual packets, and hence, network coding can possibly decrease the stability or throughput region. This is not surprising since network coding generally yields benefits for multicasting environments with multiple destinations. However, if network coding is performed in different ways, it is possible to yield enhancement to the obtained regions.

VI. Numerical Results

Numerical results illustrating the relationship between the stability and throughput regions in the cooperative multiple-access system are provided in this section. Fig. 5 plots the stability and throughput regions under scheduled access, for the channels with reception probabilities

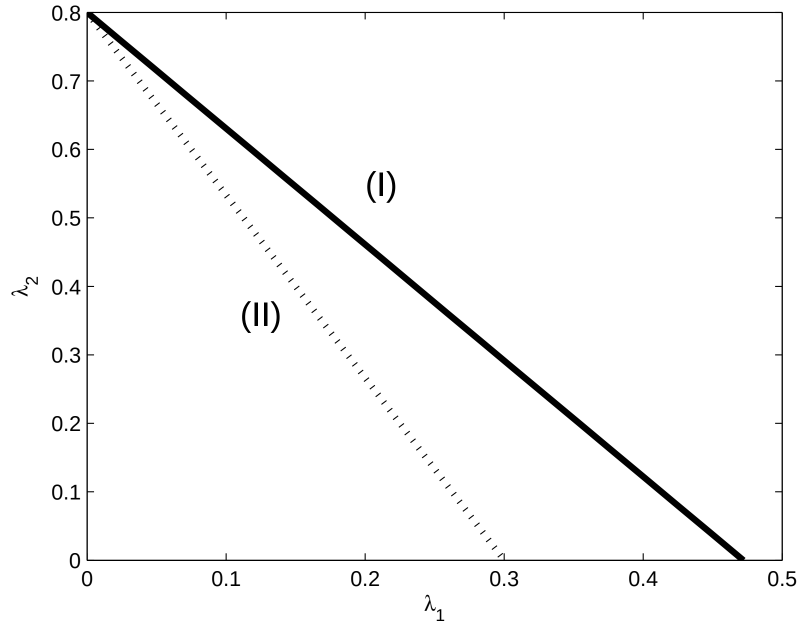

Fig. 5. Stability region and throughput regions under scheduled access. $p_{13} = 0.3$, $p_{23} = 0.8$, $p_{12} = 0.6$. (I) Stability region; throughput regions under policies 2 and 3. (II) Throughput region under policy 1.

$p_{13} = 0.3$, $p_{23} = 0.8$ and $p_{12} = 0.6$. It is seen that the relationship between the two regions is consistent with the results in Section III, that is, $L_{\mathsf{SR}} = L_2 = L_3 \supset L_1$.

The situation is more complicated in the random access case. The stability and throughput regions under random access are shown in Fig. 6 and Fig. 7. In Fig. 6, the channel reception probabilities are chosen to be $p_{13} = 0.15$, $p_{23} = 0.8$, $p_{12} = 0.7$, so that Eq. (18) is satisfied. In this case, the cooperative stability region and the throughput regions under policies 2 and 3 are observed to be the same, following the solid line; whereas the throughput region under policy 1 depicted by the dotted line is strictly bounded by the other three regions. That is, $\Re_{\mathsf{SR}} = \Re_2 = \Re_3 \supset \Re_1$.

In Fig. 7, the channel reception probabilities are chosen to be $p_{13} = 0.4$, $p_{23} = 0.7$, $p_{12} = 0.6$, so that Eq. (19) is satisfied. The boundaries of the cooperative stability region and the non-cooperative stability region (which is also the throughput region under policy 1) cross at the point $(\lambda_1, \lambda_2) = (0.2167, 0.0482)$; the throughput regions under policies 2 and 3 and the cooperative stability region are shown to confirm that $\Re_3 \supset \Re_2 \supset \Re_{\mathsf{SR}}$.

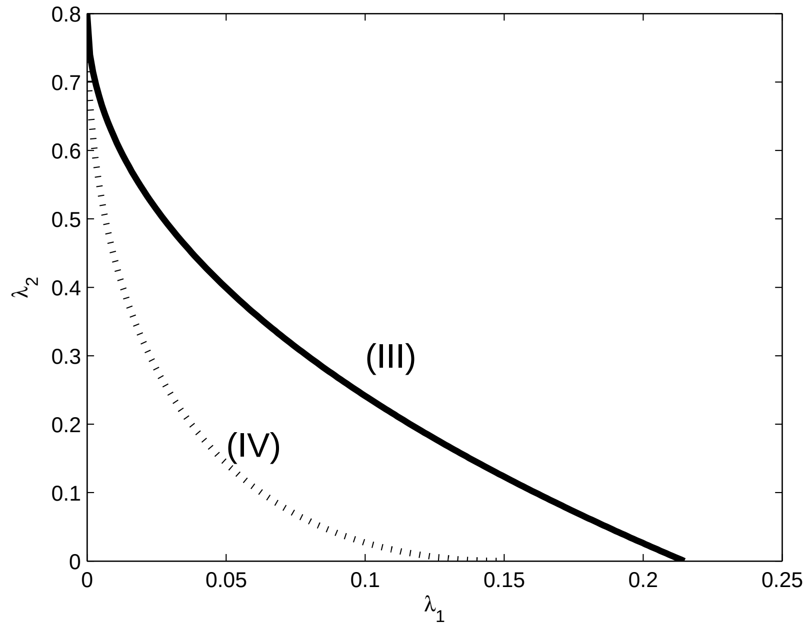

Fig. 6. Stability region and throughput regions under random access. $p_{13} = 0.15$, $p_{23} = 0.8$, $p_{12} = 0.7$. (III) Stability region; throughput regions under policies 2 and 3. (IV) Throughput region under policy 1.

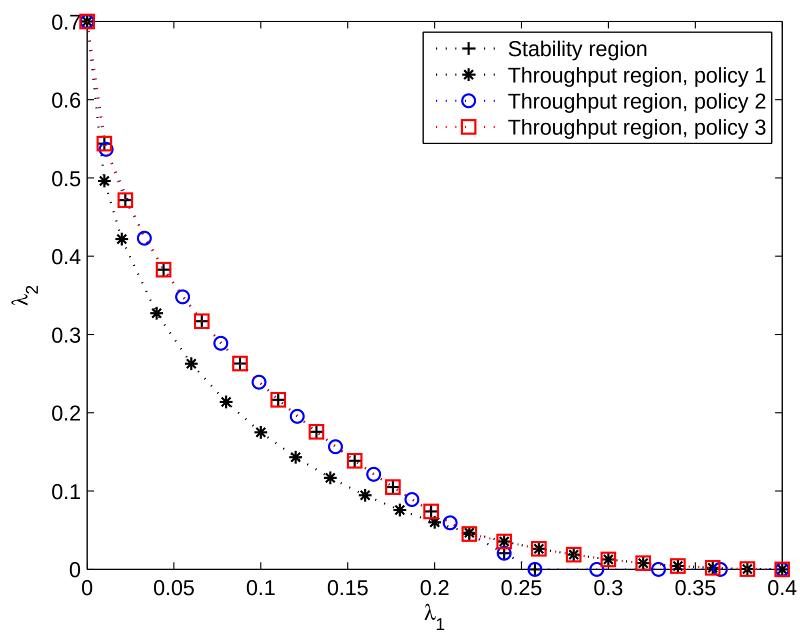

Fig. 7. Stability region and throughput regions under random access. $p_{13} = 0.4$, $p_{23} = 0.7$, $p_{12} = 0.6$.

VII. Conclusion

In this paper, we investigated the effects of "network-level" cooperation in a wireless multiple-access system with two sources unicasting traffic to a common destination over erasure channels. With a proposed cooperation strategy among the two users, we characterized both the stability region and the throughput region, under scheduled access and random access respectively. We concluded that the stability region is independent of the scheduling policies at the relay node assigned to the source packets and relayed packets; while the throughput region depends on the scheduling policies. Among a class of scheduling policies considered, we identified the two regions and determined when and whether they are identical or not. This observation demonstrates that, the addition of the extra link between the two users brings in a new dimension of complexity that affects the relationship between the stability region and throughput region in multiple-access systems.

Unlike the case of scheduled access, the cooperative stability region under random access does not always strictly contain the non-cooperative stability region, due to the lack of coordination associated with random access, which leads to collision of simultaneous transmissions.

Finally, incorporation of network coding at the relay node was considered. It turned out that this does not yield additional performance gains in this single-relay network with erasure channels.

This three-node network is a simple topology but incorporates a rich set of new factors and properties of wireless communication, so that its analysis will serve as a step toward the study of more general networks. Numerous possible extensions of this work are possible and clearly suggested, namely, inclusion of multiple intermediate nodes, diverse traffic types (unicast, multicast, broadcast) and multi-way communications.

Appendix A
A proof of Theorem 4

We start by considering a dominant system, denoted by $\mathcal{M}^1$. In system $\mathcal{M}^1$, node S transmits dummy packets with probability q_1 whenever it is empty, while node R behaves in the same way as in the original system. All the other assumptions, channel models, arrival processes and reception processes remain unaltered in the dominant system. Since the dummy packets have no contribution to the throughput but cause collision with the transmission from the other source terminal, it follows that the queue sizes in the dominant system are not smaller than the corresponding queue sizes in the original system, which implies that the stability of the dominant system is a sufficient condition for the stability of the original system.

We merge the two queues at node R, and denote the merged queue by Q_2. The two queues at node R are stable if and only if the merged queue Q_2 is stable. In the dominant system $\mathcal{M}^1$, node S always transmits with probability q_1, so the probability of success seen by Q_2 is always $q_2(1-q_1)p_{23}$, that is to say, the average service rate of Q_2 is

$$\mu_{Q_2} = q_2(1-q_1)p_{23} \tag{34}$$

To derive the stability condition for Q_2, we need to calculate its total arrival rate. There are two independent arrival processes to Q_2: first, the self-generated packets at node R that line up in queue Q_2^2, and second, the arrivals from node S, stored in queue Q_2^1. The arrival rate to Q_2^2 is λ_2; it remains to calculate the arrival rate to Q_2^1.

In system $\mathcal{M}^1$, when node R receives a dummy packet from node S, it will simply *discard* the dummy packet. When the dominant system is stable, the queue at node S is stable, so the departure rate of the source packets (dummy packets are excluded) is equal to the arrival rate to node S, which is λ_1. When node S transmits a source packet, the packet will leave node S's queue if the following two events happen together: (i) node R is empty, or node R is backlogged but decides to be silent with probability $1-q_2$, and (ii) either the destination D or node R decodes the packet, or both of them decode the packet. These two events described under (i) and (ii) are independent, and thus we have

$$\begin{aligned} &\mathbf{P}\left[\text{the transmitted packet of node } S \text{ departs } S\right] \\ =&\big\{(1-q_2)\mathbf{P}\left[Q_2 \neq 0\right] + \mathbf{P}\left[Q_2 = 0\right]\big\}\left(1-(1-p_{12})(1-p_{13})\right) \end{aligned} \tag{35}$$

Among the packets that exit node S's queue, some will exit the network if they are decoded by the destination node directly, and some of them will be relayed by node R. The latter case will happen if the following two events happen together: (i) node R decodes the packet, and (ii) the destination node D doesn't decode the packet. Therefore, we have

$$\begin{aligned} &\mathbf{P}\left[\text{the transmitted packet of node } S \text{ is relayed by node } R\right] \\ =&\big\{(1-q_2)\mathbf{P}\left[Q_2 \neq 0\right] + \mathbf{P}\left[Q_2 = 0\right]\big\}p_{12}(1-p_{13}) \end{aligned} \tag{36}$$

From Eq. (35) and Eq. (36), we obtain the conditional probability that a transmitted packet of node S (dummy packets are excluded) arrives at node R given that the transmitted packet exits node S's queue, which is

$$\begin{aligned}&\frac{\left\{(1-q_2)\mathbf{P}\left[Q_2 \neq 0\right]+\mathbf{P}\left[Q_2 = 0\right]\right\}p_{12}(1-p_{13})}{\left\{(1-q_2)\mathbf{P}\left[Q_2 \neq 0\right]+\mathbf{P}\left[Q_2 = 0\right]\right\}\left(1-(1-p_{12})(1-p_{13})\right)}\\ =&\frac{p_{12}(1-p_{13})}{1-(1-p_{12})(1-p_{13})}\end{aligned} \tag{37}$$

As we have described above, when the queue at S is stable, the departure rate of the source packets (dummy packets are excluded) is equal to the arrival source rate λ_1, so the average arrival rate to the queue Q_2^1 at node R is computed to be $\frac{p_{12}(1-p_{13})}{1-(1-p_{12})(1-p_{13})}\lambda_1$. And the total arrival rate to Q_2 is:

$$\lambda_{Q_2} = \lambda_2 + \frac{p_{12}(1-p_{13})}{1-(1-p_{12})(1-p_{13})}\lambda_1 \tag{38}$$

By Loynes' Theorem, the stability condition for the queue Q_2 at node R is given by $\lambda_{Q_2} < \mu_{Q_2}$, that is:

$$\frac{p_{12}(1-p_{13})}{1-(1-p_{12})(1-p_{13})}\lambda_1 + \lambda_2 < q_2(1-q_1)p_{23} \tag{39}$$

Then we analyze the queue Q_1 at node S in system $\mathcal{M}^1$. The arrival rate to Q_1 is λ_1; the service process of Q_1 depends on whether Q_2 at node R is empty or not: if Q_2 is empty, Q_1 receives a service rate of $q_1\left(1-(1-p_{12})(1-p_{13})\right)$; otherwise, if Q_2 is not empty, Q_1 receives a service rate of $(1-q_2)q_1\left(1-(1-p_{12})(1-p_{13})\right)$. We now need to calculate the probability that Q_2 is empty. The total arrival process to Q_2 is no longer Bernoulli, since there can be two packets arriving to R in the same time slot, one to Q_2^2, the other to Q_2^1 from Q_1. However, these two arrival processes are independent, and the inter-arrival times regarding the total arrival process remain geometrically distributed. By using the result provided in [26, Ch. 6.2], the stationary probability that Q_2 is empty still has the form of $1-\lambda_{Q_2}/\mu_{Q_2}$. As a result, the average service rate received by Q_1 is

$$\begin{aligned}\mu_{Q_1} &= \left\{(1-q_2)\mathbf{P}\left[Q_2 \neq 0\right] + \mathbf{P}\left[Q_2 = 0\right]\right\} q_1\left(1-(1-p_{12})(1-p_{13})\right) \\ &= \left(1 - \frac{\lambda_2 + \frac{p_{12}(1-p_{13})}{1-(1-p_{12})(1-p_{13})}\lambda_1}{(1-q_1)p_{23}}\right) q_1\left(1-(1-p_{12})(1-p_{13})\right) \end{aligned} \tag{40}$$

And the stability condition for Q_1 is

$$\begin{aligned}&\lambda_{Q_1} < \mu_{Q_1} \\ \Longleftrightarrow &\frac{(1-q_1)p_{23} + q_1 p_{12}(1-p_{13})}{q_1(1-q_1)p_{23}\left(1-(1-p_{12})(1-p_{13})\right)}\lambda_1 + \frac{1}{(1-q_1)p_{23}}\lambda_2 < 1\end{aligned} \tag{41}$$

The network is stable if and only if all queues in the network are stable, and from Eq. (39) and Eq. (41), the stability region of the dominant system $\mathcal{M}^1$ is characterized by:

$$\Re_{\mathcal{M}^1} = \begin{cases} \frac{(1-q_1)p_{23}+q_1p_{12}(1-p_{13})}{q_1(1-q_1)p_{23}(1-(1-p_{12})(1-p_{13}))}\lambda_1 + \frac{1}{(1-q_1)p_{23}}\lambda_2 < 1 \\ \frac{p_{12}(1-p_{13})}{1-(1-p_{12})(1-p_{13})}\lambda_1 + \lambda_2 < q_2(1-q_1)p_{23} \end{cases} \tag{42}$$

We now consider another dominant system $\mathcal{M}^2$. In system $\mathcal{M}^2$, node R transmits dummy packets with probability q_2 whenever it is empty, and node S behaves in the same way as in the original system. By following a parallel argument, we obtain the stability region of the dominant system $\mathcal{M}^2$ as:

$$\Re_{\mathcal{M}^2} = \begin{cases} \lambda_1 < q_1(1-q_2)\left(1-(1-p_{12})(1-p_{13})\right) \\ \lambda_2 + \frac{(1-q_2)p_{12}(1-p_{13})+q_2p_{23}}{(1-q_2)(1-(1-p_{12})(1-p_{13}))}\lambda_1 < q_2p_{23} \end{cases} \tag{43}$$

For a fixed transmission probability pair (q_1, q_2), the union of the two regions $\Re_{\mathcal{M}^1} \bigcup \Re_{\mathcal{M}^2}$ given by Eq. (42) and Eq. (43) provides the sufficient condition for the stability of the original system. By following the same "indistinguishability" argument as in [17], we observe that at saturation, the dominant systems and the original system become indistinguishable and hence must have the same boundary of the stability regions. As a result, the exact stability region of the original system *coincides* with the union of the stability regions of the two dominant systems, as (q_1, q_2) varies over $[0,1]^2$.

Finally, to obtain the stability region over all (q_1, q_2), we solve a constrained optimization problem as in [19]. To do this, we fix λ_1 and maximize λ_2 as (q_1, q_2) varies over $[0,1]^2$, which then generates the boundary of the stability region as a function of λ_1, λ_2 and the reception probabilities. The procedure is as follows:

We replace λ_1 by x and λ_2 by y, and the boundary of the stability region given by Eq. (42) and Eq. (43) can be written as:

$$y = (1-q_1)p_{23} - \frac{\big((1-q_1)p_{23} + q_1 p_{12}(1-p_{13})\big)x}{q_1(p_{12}+p_{13}-p_{12}p_{13})} \tag{44}$$

$$\text{for} \qquad 0 \le x \le \frac{(p_{12}+p_{13}-p_{12}p_{13})(q_2(1-q_1)p_{23}-y)}{p_{12}(1-p_{13})} \tag{45}$$

and

$$y = q_2 p_{23} - \frac{\big((1-q_2)p_{12}(1-p_{13}) + q_2 p_{23}\big)x}{(1-q_2)(p_{12}+p_{13}-p_{12}p_{13})} \tag{46}$$

$$\text{for} \qquad 0 \le x \le q_1(1-q_2)(p_{12}+p_{13}-p_{12}p_{13}) \tag{47}$$

First we consider the constrained optimization problem suggested by Eq. (44) as:

$$\max_{q_1\in[0,1]} y' = \max_{q_1\in[0,1]} \left((1-q_1)p_{23} - \frac{\big((1-q_1)p_{23} + q_1 p_{12}(1-p_{13})\big)x}{q_1(p_{12}+p_{13}-p_{12}p_{13})} \right) \tag{48}$$

Differentiating with respect to q_1 gives:

$$\frac{dy'}{dq_1} = -p_{23} + \frac{p_{23}x}{q_1^2(p_{12}+p_{13}-p_{12}p_{13})} \tag{49}$$

Setting Eq. (49) to zero yields

$$q_1^* = \sqrt{\frac{x}{p_{12}+p_{13}-p_{12}p_{13}}} \tag{50}$$

A simple calculation shows that the second derivative at q_1^* is negative, and so q_1^* must be the unique maximizer if it can be reached.

Since q_1 is a probability, we have $0 \le q_1 \le 1$; according to Eq. (50), this is equivalent to

$$0 \le x \le p_{12}+p_{13}-p_{12}p_{13} \tag{51}$$

But by Eq. (44), y must be non-negative, and so x automatically satisfies Eq. (51). Hence, by substituting Eq. (50) into Eq. (44), we obtain

$$y'_{\max} = -\frac{\big(p_{12}(1-p_{13})-p_{23}\big)x}{p_{12}+p_{13}-p_{12}p_{13}} - \frac{2p_{23}\sqrt{x}}{\sqrt{p_{12}+p_{13}-p_{12}p_{13}}} + p_{23}$$
$$\Longleftrightarrow \sqrt{\frac{x}{p_{12}+p_{13}-p_{12}p_{13}}} + \sqrt{\frac{p_{12}(1-p_{13})x}{p_{23}(p_{12}+p_{13}-p_{12}p_{13})} + \frac{y'_{\max}}{p_{23}}} = 1 \tag{52}$$

This is the boundary of the stability region provided by Eq. (44).

By applying the same procedure on Eq. (46), we can show that its boundary is also given by Eq. (52).

Therefore, the stability region of the cooperative three-node network under random access is described by

$$\Re_{\text{SR}} = \left\{ (\lambda_1, \lambda_2) : \sqrt{\frac{\lambda_1}{p_{12}+p_{13}-p_{12}p_{13}}} + \sqrt{\frac{p_{12}(1-p_{13})\lambda_1}{p_{23}(p_{12}+p_{13}-p_{12}p_{13})} + \frac{\lambda_2}{p_{23}}} < 1 \right\} \tag{53}$$

This concludes the proof of Theorem 4. We should remark here that when we construct the proof, node R is allowed to randomly pick a packet from the merged queue Q_2 to transmit (that is, either pick a packet of its own or a relayed packet from S); the analysis and derivation we presented is independent of what scheduling policy node R adopts, and hence, the stability region is independent of the scheduling policies considered and is given by Eq. (16). ■

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